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On Kaon production in e^+e^- and Semi-inclusive DIS reactions

Ekaterina Christova[1] and Elliot Leader[2]

[1] Institute for Nuclear Research and Nuclear Energy, Sofia 1784, Bulgaria
[2] Imperial College, London, UK

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Abstract. We consider semi-inclusive unpolarized DIS for the production of charged kaons and the different possibilities, both in LO and NLO, to test the conventionally used assumptions $s-\bar{s}=0$ and $D_d^{K^+-K^-}=0$. The considered tests have the advantage that they do not require any knowledge of the fragmentation functions. We also show that measurements of both charged and neutral kaons would allow the determination of the kaon FFs $D_q^{K^++K^-}$ solely from SIDIS measurements, and discuss the comparison of $(D_u-D_d)^{K^+-K^-}$ obtained independently in SIDIS and e^+e^- reactions.

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1 Introduction

It is well known that neutral current inclusive deep inelastic scattering (DIS) yields information only about quark plus antiquark parton densities. When neutrino experiments are possible one can obtain separate knowledge about quark and antiquark densities, but for the case of polarized DIS this is impossible experimentally. For this case semi-inclusive DIS (SIDIS), where some final hadron is detected, plays an essential role, but requires a knowledge of the fragmentation function (FF) for a given parton to fragment into the relevant hadron. As pointed out in [1] and more recently in [2] a precise knowledge of the FFs is vital.

When the spin state of the detected hadron is not monitored, it is possible to learn about the FFs from both $e^+e^- \to hX$ and *unpolarized* SIDIS $l+N \to lhX$. In the case of pion production $SU(2)$ plays a very helpful role in reducing the number of independent FFs needed. For kaon production, which is important for studying the strange quark densities, $SU(2)$ is less helpful, and even a combined analysis of e^+e^- and SIDIS data on both protons and neutrons does not allow an unambiguous determination of the kaon FFs [1].

It is thus conventional to make certain reasonable sounding assumptions about the strange quark densities and the kaon FFs. In this paper we discuss to what extend these assumptions can be justified and tested experimentally. In Section 2 we recall the general formulae for inclusive e^+e^- and SIDIS. In Section 3 we consider semi-inclusive $K^\pm$ production and possible tests whether, for the quark densities, $s(x)=\bar{s}(x)$, and whether, for the fragmentation functions, $D_d^{K^+}(z)=D_d^{K^-}(z)$. In Section 4 we discuss production of $K^\pm$, K_s^0 and examine possible tests for the reliability of a leading order (LO) treatment of the process.

2 General formula for e^+e^- and unpolarized SIDIS

For convenience we shall recall some general formulae for the cross sections and asymmetries in $e^+e^- \to hX$ and $e+N \to e+h+X$.

2.1 $e^+e^- \to hX$

There are two distinct measurements of interest : the total cross section $d\,\sigma_T^h(z)$ and the forward backward (FB) asymmetry A_{FB}^h. If $d^2\sigma^h/(dz\,d\cos\theta)$ is the differential cross section for $e^+e^- \to hX$, these quantities are defined as:

$$d\,\sigma_T^h(z) = \int_{-1}^{+1}\left(\frac{d^2\sigma^h}{dz\,d\,\cos\theta}\right)d\cos\theta \qquad (1)$$

$$A_{FB}^h(z) = \left[\int_{-1}^{0} - \int_{0}^{+1}\right]\left(\frac{d^2\sigma^h}{dz\,d\,\cos\theta}\right)d\cos\theta, \qquad (2)$$

where θ is the CM scattering angle and z is, neglecting masses, the fraction of the momentum of the fragmenting parton transferred to the hadron h: $z=2(P^h.q)/q^2=E^h/E$, where E^h and E are the CM energies of the final hadron h and the initial lepton.

From CP invariance it follows that

$$d\,\sigma_T^h(z) = d\,\sigma_T^{\bar{h}}(z), \qquad A_{FB}^h(z) = -\,A_{FB}^{\bar{h}}(z), \qquad (3)$$

where $\bar{h}$ is the C-conjugate of the hadron h. Eq. (3) implies that the total cross section $d\sigma_T^h$ actually provides information only about $D_q^{h+\bar{h}} \equiv D_q^h + D_q^{\bar{h}}$, while measurement of A_{FB}^h determines the non-singlet (NS) combinations $D_q^{h-\bar{h}} \equiv D_q^h - D_q^{\bar{h}}$, and this is true in all orders of QCD.

In LO the formula are especially simple:

$$d\sigma_T^h(z) = 3\,\sigma_0 \sum_q \hat{e}_q^2\, D_q^{h+\bar{h}}, \quad \sigma_0 = \frac{4\pi\alpha_{em}^2}{3\,s} \tag{4}$$

$$A_{FB}^h(z) = 3\,\sigma_0 \sum_q \frac{3}{2}\,\hat{a}_q\, D_q^{h-\bar{h}}. \tag{5}$$

Assuming both photon and Z^0-boson exchange we have:

$$\begin{aligned}\hat{e_q}^2(s) &= e_q^2 - 2e_q\, v_e\, v_q\, \Re e\, h_Z + \\ &\quad +(v_e^2+a_e^2)\,\left[(v_q)^2+(a_q)^2\right]\,|h_Z|^2 \\ \hat{a}_q &= 2\,a_e\,a_q\,\left(-e_q\,\Re e\, h_Z + 2\,v_e\,v_q\,|h_Z|^2\right),\end{aligned} \tag{6}$$

where $h_Z = [s/(s-m_Z^2+im_Z\Gamma_Z)]/\sin^2 2\theta_W$. In (6) e_q is the charge of the quark q in units of the proton charge, and, as usual,

$$\begin{aligned} v_e &= -1/2 + 2\sin^2\theta_W, \quad a_e = -1/2, \\ v_q &= I_3^q - 2e_q\sin^2\theta_W, \quad a_q = I_3^q, \\ & I_3^u = 1/2, \quad I_3^d = -1/2. \end{aligned} \tag{7}$$

2.2 Unpolarized SIDIS $e + N \to e + h + X$

In semi-inclusive deep inelastic scattering, we consider the non-singlet difference of cross-sections $\sigma_N^{h-\bar{h}}$, where the measurable quantity is the ratio:

$$R_N^{h-\bar{h}} = \frac{\sigma_N^{h-\bar{h}}}{\sigma_N^{DIS}}, \qquad \sigma_N^{h-\bar{h}} = \sigma_N^h - \sigma_N^{\bar{h}}. \tag{8}$$

For simplicity, we use $\tilde{\sigma}_N^h$ and $\tilde{\sigma}_N^{DIS}$ in which common kinematic factors have been removed:

$$\tilde{\sigma}_N^h \equiv \frac{x(P+l)^2}{4\pi\alpha^2}\left(\frac{2y^2}{1+(1-y)^2}\right)\frac{d^3\sigma_N^h}{,}dxdydz \tag{9}$$

$$\tilde{\sigma}_N^{DIS} \equiv \frac{x(P+l)^2}{4\pi\alpha^2}\left(\frac{2y^2}{1+(1-y)^2}\right)\frac{d^2\sigma_N^{DIS}}{dxdy}. \tag{10}$$

Here P and l are the nucleon and lepton four momenta, and x, y, z are the usual deep inelastic kinematic variables: $x = Q^2/2P.q = Q^2/2M\nu$, $y = P.q/P.l = \nu/E$, $z = P.P_h/P.q = E^h/\nu$, where E and E^h are the lab. energies of the incoming lepton and final hadron. Note that, both in e^+e^- and in SIDIS, neglecting masses, z always measures the fraction of the parton momentum transferred to the produced hadron.

Since the kinematic factors for σ_N^h and σ_N^{DIS} are the same, we can write:

$$\tilde{\sigma}_N^{h-\bar{h}} = R_N^{h-\bar{h}}\tilde{\sigma}_N^{DIS}. \tag{11}$$

If we work in LO we have:

$$\tilde{\sigma}_N^{DIS} = \sum_{q,\bar{q}} e_q^2\, q_i(x,Q^2), \tag{12}$$

whereas in NLO:

$$\tilde{\sigma}_N^{DIS} = \sum_{q,\bar{q}} e_q^2 \left\{ q_i\left(1+\frac{\alpha_s}{2\pi}\otimes C_q\right) + \frac{\alpha_s}{2\pi} g\otimes C_g\right\}. \tag{13}$$

As shown in [3], the general expression for the cross section differences, in NLO, is given by:

$$\begin{aligned}\tilde{\sigma}_p^{h-\bar{h}}(x,z) &= \frac{1}{9}\left[4u_V\otimes D_u^{h-\bar{h}} + d_V\otimes D_d^{h-\bar{h}}\right. \\ &\quad \left. +(s-\bar{s})\otimes D_s^{h-\bar{h}}\right]\otimes\hat{\sigma}_{qq}(\gamma q\to qX), \\ \tilde{\sigma}_n^{h-\bar{h}}(x,z) &= \frac{1}{9}\left[4d_V\otimes D_u^{h-\bar{h}} + u_V\otimes D_d^{h-\bar{h}}\right. \\ &\quad \left. +(s-\bar{s})\otimes D_s^{h-\bar{h}}\right]\otimes\hat{\sigma}_{qq}(\gamma q\to qX). \end{aligned} \tag{14}$$

Here $\hat{\sigma}_{qq}$ is the perturbatively calculable, hard partonic cross section $q\gamma^* \to q + X$:

$$\hat{\sigma}_{qq} = \hat{\sigma}_{qq}^{(0)} + \frac{\alpha_s}{2\pi}\hat{\sigma}_{qq}^{(1)}, \tag{15}$$

normalized so that $\hat{\sigma}_{qq}^{(0)} = 1$.

It is seen that $\tilde{\sigma}_N^{h-\bar{h}}$ involves only NS parton densities and fragmentation functions, implying that its Q^2 evolution is relatively simple. Eq.(14) is sensitive to the valence quark densities, but also to the completely unknown combination $(s-\bar{s})$. The term $(s-\bar{s})D_s^{h-\bar{h}}$ plays no role in Pion production, since, by SU(2) invariance, $D_s^{\pi^+-\pi^-} = 0$. However it is important for kaon production, for which $D_s^{K^+-K^-}$ is a favoured transition, and thus expected to be big.

Up to now all analyses of experimental data have assumed $s = \bar{s}$. In the next Sections we shall consider the production of kaons, $h = K^\pm$ and $h = K^\pm, K_s^0$ and show how this assumption, and the assumption $D_d^{K^+-K^-} = 0$, can be tested without requiring knowledge of the FFs.

3 Production of charged kaons

As seen from (14), in $R_N^{K^+-K^-}$ both $s-\bar{s}$ and $D_d^{K^+-K^-}$ appear. They are expected to be small, and the usual assumption is that they are equal to zero. Here we examine to what extent one can test these assumptions experimentally in SIDIS.

It was shown in [3], that even if we combine data on the forward-backward asymmetry $A_{FB}^{K^+-K^-}$ in e^+e^--annihilation with measurements of K^+ and K^- production in SIDIS, we cannot determine the fragmentation functions without assumptions. The reason is that we have 3 measurements for the 4 unknown quantities $D_{u,d,s}^{K^+-K^-}$ and $(s-\bar{s})$. Thus, one needs an assumption: either $s-\bar{s} = 0$

or $D_d^{K^+-K^-} = 0$. In fact, up to now, all analyses of experimental data have been performed assuming both $s-\bar{s}=0$ and $D_d^{K^+-K^-} = 0$.

Note, that from the quark content of $K^\pm$, the assumption $D_d^{K^+-K^-} = 0$ seems very reasonable if the $K^\pm$ are directly produced. However, if they are partly produced via resonance decay this argument is less persuasive. Of course $e^+e^- \to K^\pm X$ sheds no light on this issue.

3.1 LO approximation

In **LO** we have:

$$\tilde{\sigma}_p^{K^+-K^-} = \frac{1}{9}[4\, u_V\, D_u^{K^+-K^-} + d_V\, D_d^{K^+-K^-} + (s-\bar{s})\, D_s^{K^+-K^-}], \qquad (16)$$

$$\tilde{\sigma}_n^{K^+-K^-} = \frac{1}{9}[4\, d_V\, D_u^{K^+-K^-} + u_V\, D_d^{K^+-K^-} + (s-\bar{s})\, D_s^{K^+-K^-}]. \qquad (17)$$

From a theoretical point of view it is more useful to consider the following combinations of cross-sections, which, despite involving differences of cross-sections, are likely to be large:

$$(\tilde{\sigma}_p - \tilde{\sigma}_n)^{K^+-K^-} = \frac{1}{9}[(u_V - d_V)\,(4D_u - D_d)^{K^+-K^-}],$$
$$(\tilde{\sigma}_p + \tilde{\sigma}_n)^{K^+-K^-} = \frac{1}{9}[(u_V + d_V)\,(4D_u + D_d)^{K^+-K^-} + 2(s-\bar{s}) D_s^{K^+-K^-}]. \qquad (18)$$

We define:

$$R_+(x,z) \equiv \frac{(\tilde{\sigma}_p + \tilde{\sigma}_n)^{K^+-K^-}}{u_V + d_V},$$
$$R_-(x,z) \equiv \frac{(\tilde{\sigma}_p - \tilde{\sigma}_n)^{K^+-K^-}}{u_V - d_V}. \qquad (19)$$

From a study of the x and z dependence of these we can deduce the following:

1) if $R_-(x,z)$ is a function of z only, then this suggests that a LO approximation is reasonable.

2) if $R_+(x,z)$ is *also* a function of z only, then, since $D_s^{K^+-K^-}$ is a favoured transition, we can conclude that $(s-\bar{s})=0$.

3) if $R_+(x,z)$ and $R_-(x,z)$ are *both* functions of z only, and if in addition, $R_+(x,z) = R_-(x,z)$, then both $s-\bar{s}=0$ *and* $D_d^{K^+-K^-} = 0$.

4) if $R_+(x,z)$ and $R_-(x,z)$ are *both* functions of z only, but they are *not* equal, $R_+(x,z) \neq R_-(x,z)$, we conclude that $s-\bar{s}=0$, *but* $D_d^{K^+-K^-} \neq 0$.

5) if $R_-(x,z)$ is not a function of z only, then NLO corrections are needed, which we consider below.

3.2 NLO approximation

If an NLO treatment is necessary it is still possible to reach some conclusions, though less detailed than in the LO case. We now have:

$$(\tilde{\sigma}_p - \tilde{\sigma}_n)^{K^+-K^-} = \frac{1}{9}(u_V - d_V) \otimes (1 + \alpha_s\, C_{qq}) \otimes (4D_u - D_d)^{K^+-K^-} \qquad (20)$$

$$(\tilde{\sigma}_p + \tilde{\sigma}_n)^{K^+-K^-} = \frac{1}{9}\Big[(u_V + d_V) \otimes (4D_u + D_d)^{K^+-K^-} + 2(s-\bar{s}) \otimes D_s^{K^+-K^-}\Big] \otimes (1 + \alpha_s\, C_{qq}). \qquad (21)$$

Suppose we try to fit both (20) and (21) with one and the same fragmentation function. If this is successful, then, since u_V and d_V and the Wilson coefficients are accurately known and $D_s^{K^+-K^-}$ is favoured, we can conclude that both $s-\bar{s}=0$ *and* $D_d^{K^+-K^-} = 0$, i.e.

$$(\tilde{\sigma}_p - \tilde{\sigma}_n)^{K^+-K^-} \approx \frac{4}{9}(u_V - d_V) \otimes (1 + \alpha_s\, C_{qq}) \otimes D_u^{K^+-K^-}, \qquad (22)$$

$$(\tilde{\sigma}_p + \tilde{\sigma}_n)^{K^+-K^-} \approx \frac{4}{9}(u_V + d_V) \otimes (1 + \alpha_s\, C_{qq}) \otimes D_u^{K^+-K^-}. \qquad (23)$$

Note that for all above tests, both in LO and NLO approximation, we don't require a knowledge of $D_{u,d}^{K^+-K^-}$. This is especially important since the e^+e^- total cross section data determine only the $D_q^{K^++K^-}$, and these are relatively well known, while $D_{u,d}^{K^+-K^-}$ can be determined solely from A_{FB} in e^+e^- or from SIDIS.

The results of the above tests would indicate what assumptions are reliable in trying to extract the fragmentation functions $D_{u,d,s}^{K^\pm}$ from the same data.

4 Production of charged and neutral kaons

The description of SIDIS and e^+e^- reactions, in which one monitors neutral $K_s^0 = (K^0 + \bar{K}^0)/\sqrt{2}$ as well as charged $K^\pm$ does not require any further FFs. This is due to SU(2) invariance which relates the neutral to the charged kaon FFs:

$$D_u^{K^++K^-} = D_d^{K^0+\bar{K}^0}, \qquad D_d^{K^++K^-} = D_u^{K^0+\bar{K}^0},$$
$$D_s^{K^++K^-} = D_s^{K^0+\bar{K}^0}.$$

In principle this allows the determination of the kaon FFs $D_{u,d,s}^{K^++K^-}$ solely from SIDIS measurements, without the problem of combining e^+e^- data and SIDIS data at widely different value of Q^2.

Two possible measurements can be performed: with $(K^+ + K^- - 2\,K_s^0)$ and with $(K^+ + K^- + 2\,K_s^0)$.

4.1 The combination $K^+ + K^- - 2K_s^0$

In LO, we have for e^+e^-

$$d\sigma_T^{K^+ + K^- - 2K_s^0}(z) = = 3\,\sigma_0 \left[(\hat{e}_u^2 - \hat{e}_d^2)_{m_Z^2}(D_u - D_d)^{K^+ + K^-}\right], \quad (24)$$

$$d\sigma_T^{K^+ + K^- - 2K_s^0} \equiv d\sigma_T^{K^+} + d\sigma_T^{K^-} - 2\,d\sigma_T^{K_s^0}, \quad (25)$$

and for SIDIS

$$d\tilde{\sigma}_p^{K^+ + K^- - 2K_s^0}(x,z) = = \frac{1}{9}\,[4(u+\bar{u}) - (d+\bar{d})](D_u - D_d)^{K^+ + K^-}, \quad (26)$$

$$d\tilde{\sigma}_n^{K^+ + K^- - 2K_s^0}(x,z) = = \frac{1}{9}\,[4(d+\bar{d}) - (u+\bar{u})](D_u - D_d)^{K^+ + K^-}. \quad (27)$$

Thus, due to SU(2) -invariance all three processes measure the same NS fragmentation function $(D_u - D_d)^{K^+ + K^-}$, whose evolution does not involve the very poorly known gluon fragmentation functions. Note that the combinations of quark densities in the above do have a singlet component, but that is not a problem. Note, however, that $s + \bar{s}$ does not enter, and this is true in NLO as well.

Tests for whether LO is a reasonable approximation for the SIDIS reactions can be made as follows. In LO one should have:

1) for proton targets

$$\frac{d\tilde{\sigma}_p^{K^+ + K^- - 2K_s^0}(x,z)}{4(u+\bar{u}) - (d+\bar{d})} = \text{function of } z \text{ only} \equiv \equiv f_p(z) = (D_u - D_d)^{K^+ + K^-}(z) \quad (28)$$

2) for neutron targets

$$\frac{d\tilde{\sigma}_n^{K^+ + K^- - 2K_s^0}(x,z)}{4(d+\bar{d}) - (u+\bar{u})} = \text{function of } z \text{ only} \equiv \equiv f_n(z) = (D_u - D_d)^{K^+ + K^-}(z), \quad (29)$$

where the PD's are determined in LO, see for example [4].

3) and if measurements for both proton and neutron targets are available, then also

$$f_p(z) = f_n(z) \quad (30)$$

should hold, as expected from (28) and (29).

The above LO-tests do not require knowledge of the FFs. Concerning the measurement of FFs, an attempt was made in [1] to combine data on e^+e^- and SIDIS. The evolution involved there required an estimate of the gluon FF which induced quite large errors. In the present case, we study the NS combination $(D_u - D_d)^{K^+ + K^-}$, which can be measured both in e^+e^- and SIDIS , (24)-(27), and whose evolution in Q^2 is straightforward since it does not involve the gluon FFs.

Consequently one could try to combine e^+e^- data at $Q^2 \simeq m_Z^2$, where Z^0-exchange is the dominant contribution, with SIDIS experiments at $Q^2 << m_Z^2$ where γ-exchange dominates. For example one could test whether

$$\frac{9\,d\tilde{\sigma}_p^{K^+ + K^- - 2K_s^0}(x,z,Q^2)}{d\sigma_T^{K^+ + K^- - 2K_s^0}(z, m_Z^2)_{\downarrow Q^2}} = = \frac{[4(u+\bar{u}) - (d+\bar{d})](x,Q^2)}{3\,\sigma_0\,(\hat{e}_u^2 - \hat{e}_d^2)_{m_Z^2}}. \quad (31)$$

Here $d\sigma_T^{K^+ + K^- - 2K_s^0}(z, m_Z^2)_{\downarrow Q^2}$ denotes that the data is measured at m_Z^2 and then evolved to Q^2 according to the DGLAP equations. This would be a test of LO, but also a test of the factorization into parton densities times FFs.

Note that it is essential to form the difference of cross sections $K^+ + K^- - 2K_s^0$, involving neutral kaons, in order to eliminate the s-quark contributions.

4.2 The combination $K^+ + K^- + 2K_s^0$

In LO we have (using for brevity the notation $(K) \equiv K^+ + K^- + 2K_s^0$),

for e^+e^-:

$$d\sigma_T^{(K)}(z) = 3\,\sigma_0 \left[(\hat{e}_u^2 + \hat{e}_d^2)_{m_Z^2}(D_u + D_d)^{K^+ + K^-} + +2\,\hat{e}_d^2 D_s^{K^+ + K^-}\right], \quad (32)$$

and for SIDIS:

$$\tilde{\sigma}_p^{(K)}(x,z,Q^2) = \frac{1}{9}\left[(4(u+\bar{u}) + +(d+\bar{d}))(D_u + D_d)^{K^+ + K^-} + 2(s+\bar{s})D_s^{K^+ + K^-}\right],$$

$$\tilde{\sigma}_n^{(K)}(x,z,Q^2) = \frac{1}{9}\left[(4(d+\bar{d}) + +(u+\bar{u}))(D_u + D_d)^{K^+ + K^-} + 2(s+\bar{s})D_s^{K^+ + K^-}\right] \quad (33)$$

From these expressions we can form the following test of the LO approximation in these processes.

1) In LO we have:

$$\frac{3\,(\tilde{\sigma}_p - \tilde{\sigma}_n)^{K^+ + K^- + 2K_s^0}(x,z)}{\left(u + \bar{u} - (d+\bar{d})\right)(x)} = \text{function of } z \text{ only} = = (D_u + D_d)^{K^+ + K^-}(z). \quad (34)$$

2) If K_s^0 are not measured, LO would be a good approximation if:

$$\frac{9(\tilde{\sigma}_p - \tilde{\sigma}_n)^{K^+ + K^-}(x,z)}{\left(u + \bar{u} - (d+\bar{d})\right)(x)} = \text{function of } z \text{ only} = = (4D_u - D_d)^{K^+ + K^-}(z), \quad (35)$$

i.e. only the combination of FFs on the r.h.s. is different from (34).

In summary, if in addition to charged $K^\pm$, also neutral K_s^0 are measured, we can 1) determine all FFs, $D_{u,d,s}^{K^++K^-}$, solely from SIDIS, i.e. it is not necessary to use data from e^+e^-- performed at very different Q^2 , and 2) we can compare the thus obtained FFs for the non-singlet $(D_u - D_d)^{K^++K^-}$ with those measured in e^+e^- – eq.(24), since for these Q^2-evolution is straightforward.

Of course the above results are true only in LO, i.e. if the LO tests (28)-(35) hold. Only in this case can we combine measurements on SIDIS $\tilde{\sigma}_N^{K^++K^-\pm 2K_s^0}$ and $\sigma_T^{K^++K^--2K_s^0}$ from e^+e^- in a simple way. For neither of these tests is a knowledge of the FFs necessary.

5 Conclusions

We have considered possible tests for $s-\bar{s} = 0$ and $D_d^{K^+-K^-} = 0$ in unpolarized SIDIS with final charged $K^\pm$, both in LO and NLO in QCD. We show that, if in addition to $K^\pm$ also the neutral K_s^0 are measured, different possibilities to test the LO approximation and the factorization into PDs and FFs appear. For some of these tests SIDIS is enough, for others the combined data of the total cross section in e^+e^- scattering in addition to SIDIS is also needed. In all proposed tests no knowledge of the FFs is necessary.

In our approach we consider the sum and difference of cross sections for hadron h and its C-conjugate $\bar{h}$. The cross section differences , $h - \bar{h}$, are NS and, both their Q^2-evolution and NLO corrections in QCD are straightforward, since they don't mix with other PDs or FFs. But they involve poorly known quantities such as the non-singlets $s - \bar{s}$ and $D_d^{K^+-K^-}$, and we suggest some tests for these quantities. Quite the opposite is true when the sum of cross sections $h + \bar{h}$ is considered. In this case the Q^2-evolution and NLO corrections involve the poorly known gluon FFs, but the cross sections contain the best known combinations of PDs $q + \bar{q}$, measured in DIS, and $D^{h^++h^-}$ measured in e^+e.

We have tried to exploit some of the advantages of both types of combinations of data.

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